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COMPRESSIBLE FLOW THROUGH A POROUS PLATE

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Abstract—A simple one-dimensional theory is given for the steady, compressible, adiabatic flow of a perfect gas through a porous plate. The Dupuit–Forcheimer relation, valid for incompressible flow, is replaced by an isentropic compression when the gas enters the plate and a non-isentropic sudden enlargement process when it exits. Darcy's equation is used in a form applicable to compressible adiabatic flow. A consequence of this study is that the Mach number at the downstream surface may be much smaller than unity, even when the flow through the plate is choked. As the pressure at the downstream surface of the plate decreases, the flow remains choked, but the downstream Mach number increases. For a sufficiently small downstream pressure this Mach number will be greater than unity. A wide range of downstream Mach numbers from subsonic to supersonic is thus possible, even though the flow is choked. For incompressible flow, the volumetric flow rate is usually determined by the pressure differential across the plate. The equivalent compressible relation is shown to consist of a plot of upstream Mach number versus the pressure ratio across the plate. The incompressible result can also be shown on this plot; it differs from the compressible one, except when the plate is thick.

NOMENCLATURE

b,	drag coefficient;
B_0,	Darcy's constant;
d,	average pore diameter;
f,	function defined by equation (12);
F, G_i,	Mach number functions;
h,	enthalpy;
L,	porous plate thickness;
M,	Mach number;
p,	pressure;
R,	gas constant;
R_d,	Reynolds number;
s,	entropy;
T,	temperature;
u,	velocity;
x,	distance in the direction of flow;
X,	drag.

Greek symbols

β,	porosity;
γ,	ratio of specific heats;
η,	viscosity;
ρ,	density.

INTRODUCTION

For the vast majority of problems concerning flow of a liquid in a porous medium, the fluid is considered incompressible (or slightly compressible) and the convective terms are neglected. For most problems these assumptions are not unreasonable, and they have been used in investigations of this type [1, 2]. When the fluid is a compressible gas subject to large pressure gradients, such assumptions are no longer valid, and a different methodology must be applied. The purpose of this paper is to develop a simple theory for one-dimensional, compressible flow of a gas through a porous plate.

Flows outside the classical seepage regime are usually dealt with by modifying Darcy's equation, either by using a power-law velocity dependence or by the addition of more terms. An example of the later approach is the excellent paper by Green and Duwez [3]. Before presenting our approach, we describe [3] in some detail to contrast our sharply different point of view with theirs. The experiments described in [3] range from small mass flow rates where Darcy's equation is valid, to quite large rates where compressibility ought to be important. Nevertheless, in their analysis they explicitly omit compressibility, regardless of the mass flow rate. Instead, an additional term proportional to ρu^2 is included in Darcy's equation, where ρ and u are density and velocity, respectively. Two empirical constants thus appear in their form of Darcy's equation where both constants depend, in part, on the plate's porosity, i.e. its void ratio. Experimental results are presented dimensionally in terms of a graph of pressure-square gradient vs. mass flow rate.

In opposition to the foregoing approach, we find that compressibility should be important when the mass flow rate (and hence the pressure ratio across the plate) is large. The most obvious phenomena to expect in this circumstance is choking when the mass flow rate through the plate reaches a maximum. As in nozzle flow, the most unambiguous way of demonstrating the onset of choking is to vary the pressure downstream of the plate (equivalent to varying nozzle exit pressure) while the upstream pressure is kept fixed. Although Green and Duwez [3] do not state if they did this, it seems probable that only the upstream pressure was varied. This approach, in conjunction with the type of plot used for their results, would prevent them from detecting the onset of choking. Finally, they did not consider surface effects, which we will show are important when the mass flow rate is large.

In order to concentrate more fully on the gas–solid interaction, the following simplifying assumptions are here introduced:

1. Steady, one-dimensional flow is assumed.
2. The physical model of a gas flowing in a straight, frictionless, insulated duct of constant cross section is adopted. Situated in the duct is a porous plate of uniform thickness (see Fig. 1).
3. A perfect gas is assumed with a constant value for the ratio of specific heats.
4. Continuum mechanics govern the flow.
5. Heat transfer in the plate is neglected.

We take as a mathematical model for the viscous flow inside the plate the formulation applicable to flow in a straight duct with friction. This description is intended to represent average flow conditions at any cross section of the plate and does *not* imply the assumption of a capillary model in which the pores are straight ducts transverse to the plate. The theory associated with this approach is well established and widely known. (For example, see Shapiro [4], whose work is heavily relied upon here.) We follow [4] and use the Mach number as the independent variable, which yields a formulation that is physically elegant and mathematically simple. An important result of this approach is that momentum considerations can be dealt with separately, as has been done in the subsequent analysis.

A key feature of this analysis is the generalization of the Dupuit–Forcheimer relation [1, 2], which says that the volumetric flow rate is constant across the surface of a porous medium, i.e. the ratio of external velocity to pore velocity is the porosity. The fluid thus adjusts to the smaller flow area available to it in the medium. In incompressible flow theory, this relation is used regardless of whether the fluid is entering or leaving the medium. Instead, we use the isentropic assumption when the fluid is entering the plate. On exiting, the flow is assumed to adjust to the larger available area by means of an irreversible process.

The most important consequence of this analysis is that although the flow may be choked, the Mach number downstream of the plate is generally not unity. Specifically, as the downstream pressure decreases, the flow becomes choked, but the downstream Mach number may still be much smaller than unity. As the pressure further decreases, the flow remains choked, while the downstream Mach number increases. In fact, this Mach number will become greater than unity for a sufficiently small downstream pressure. Although the flow through the plate is choked, a wide range of downstream Mach numbers ranging from subsonic to supersonic is therefore possible. In addition, a limiting pressure ratio and Mach number exist beyond which solutions are no longer possible according to this theory.

Results of this work can be given as a plot of upstream Mach number vs. the pressure ratio across the plate. The incompressible result can also be shown on our plot; it differs from the compressible one, except when all Mach numbers are small or when the plate is thick.

We formulate the problem in the following section. The case when choking does not occur is analyzed first. Momentum considerations are then dealt with in a separate section. The subsequent section treats the choked-flow case, while the final section discusses a number of important aspects of the theory. Finally, we note that this paper is a condensed version of [5].

FORMULATION

Consider one-dimensional flow of a perfect gas through a plate of uniform porosity β. The cross-sectional area of the plate is taken to be unity; consequently, the area available to the flow within the plate is β, which must be less than unity. A streamtube of fluid contracts in area as it enters the plate and expands in area as it leaves. These changes, referred to as surface effects in the Introduction, occur between locations 1 and 2, and 3 and 4, as shown in Fig. 1. Both area changes are assumed to occur in a short distance in comparison to the plate's thickness.

Upstream of the plate, the flow is at a low subsonic Mach number given by

$$M_1^2 = \frac{u_1^2}{\gamma(p_1/\rho_1)} = \frac{RT_1(\rho_1 u_1)^2}{\gamma p_1^2}, \tag{1}$$

where γ is the (constant) ratio of specific heats, p is pressure, R is the gas constant and T is temperature. Because $M_1 \ll 1$, the thermodynamic quantities in equation (1) differ negligibly from their stagnation values. Similarly, in all ensuing formulas we use without further statement or justification the approximation $[1 + (\gamma - 1)M_1^2/2] \cong 1$.

Between locations 1 and 2 we assume the area change is accomplished by an isentropic process; the flow is thus analogous to that in the convergent part of a nozzle. Associated with the flow is a small loss of stagnation pressure, but this loss is minute compared with that between locations 2 and 3 and therefore is neglected.

Between locations 2 and 3 we assume the flow is adiabatic but non-isentropic due to viscous-energy dissipation.† Flow conditions are described by a Fanno curve, which combines the adiabatic assumption with continuity. Such a curve is independent not only of the form of the drag term in the momentum equation, but of the momentum equation itself. A fundamental property of a Fanno curve is that at 3′ or 4* (see Fig. 2) the entropy is a maximum and the Mach number is unity. Associated with this is the phenomenon of choking. Since $M_2 < 1$, we have the condition $M_3 \leqslant 1$, which plays an important role in the subsequent analysis.

Between 3 and 4 we assume the area change is accomplished by a sudden enlargement, which is an adiabatic but non-isentropic process. When $M_3 < 1$, this is a compression, $p_3 < p_4$, which results in $M_4 < M_3$. When M_3 is near unity and β is small, an entropy increase occurs which cannot be neglected. Since this case is

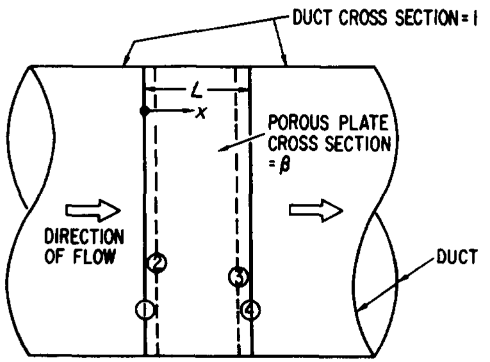

Fig. 1. Flow schematic for a porous plate inside a duct.

† We could also assume the flow between 2 and 3 to be isothermal. However, [4] (Chapter 6) shows that both assumptions yield similar results qualitatively and quantitatively. For experiments of long duration involving flow through a thin plate situated in an insulated duct, we adopt the adiabatic assumption as the more realistic one.

important, we use the more exact irreversible process throughout the analysis, even though an isentropic process is feasible when M_3 is small.

These considerations are conveniently illustrated by a Mollier diagram (Fig. 2). The ordinate is the enthalpy h, where the stagnation enthalpy h_0 is constant from 1 to 4, since the entire process is adiabatic. Locations 1, 2, 3 and 4 are state points on the diagram and hereafter are so designated. States 1 and 4 are connected by a Fanno curve, since these states have the same mass flow rate and h_0. This curve is to the right of the Fanno curve passing through 2 and 3 because of the smaller mass flow per unit area. When $\beta = 1$ both curves coincide; as β decreases from unity they diverge. Flow conditions between 1 and 2 are given by an isentropic subsonic expansion, between 2 and 3 are given by the Fanno curve, while the irreversible subsonic compression occurs between 3 and 4.

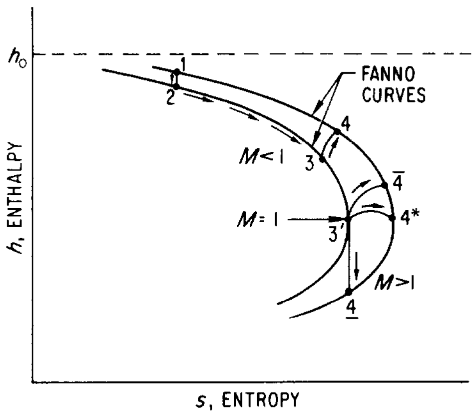

FIG. 2. Mollier diagram for flow through a porous plate.

The process between states 3 and 4 is valid when $M_3 < 1$ and in the limit $M_3 \to 1$. When $M_3 = 1$, i.e. states 3 and 3′ coincide, the flow chokes and the analysis must be altered to allow for other solutions in addition to the $M_3 \to 1$ limit. We shall first consider the $M_3 < 1$ case, along with the limit $M_3 \to 1$.

$M_3 < 1$ AND $M_3 \to 1$ ANALYSIS

States 1 and 4 are related by the Fanno curve [4]

$$M_1^2 = \left(\frac{p_4}{p_1}\right)^2 G_0(M_4^2), \tag{2a}$$

where

$$G_0(M^2) \equiv M^2[1 + (\gamma - 1) M^2/2].$$

States 1 and 2 are isentropically related by [4]

$$M_1^2 = \beta^2 G_1(M_2^2), \tag{2b}$$

where

$$G_1(M^2) \equiv \frac{M^2}{[1 + (\gamma - 1) M^2/2]^{(\gamma+1)/(\gamma-1)}}.$$

States 3 and 4 are related by the momentum equation for a sudden enlargement from an area β to unit area. This equation is ([4], problem 5.23)

$$(1 - \beta) p_3 + (\beta)(p_3 + \rho_3 u_3^2) = (1)(p_4 + \rho_4 u_4^2), \tag{3}$$

which is valid only when $M_3 < 1$, or in the limit $M_3 \to 1$. By means of equations (2), the isentropic relation for p_2/p_1, and the Fanno curve relation for p_2/p_3, equation (3) can be written as

$$\frac{M_4^2 + (\gamma - 1) M_4^4/2}{(1 + \gamma M_4^2)^2} = \frac{\beta^2 M_3^2 + (\gamma - 1)\beta^4 M_3^4/2}{(1 + \gamma\beta M_3^2)^2}. \tag{2c}$$

The three equations (2) relate four Mach numbers, and consequently an additional relation is necessary. This is provided by conservation of momentum between states 2 and 3, which is given later.

An upper bound on M_4, obtained from equation (2c) and the condition $M_3 < 1$, is

$$\frac{G_0(M_4^2)}{(1 + \gamma M_4^2)^2} < \frac{\gamma + 1}{2}\left(\frac{\beta}{1 + \gamma\beta}\right)^2. \tag{4}$$

Hence, $M_4 \leqslant \overline{M}_4$, where $\overline{M}_4$ is obtained from (4) by replacing the inequality by an equality sign. Figure 3 shows $\overline{M}_4^2$ vs. β for $\gamma = (7/5)$. From equation (2a) and inequality (4), we see that $(p_1 M_1/p_4)^2$ also has an upper bound,

$\overline{(p_1M_1/p_4)}^2$. In the limit $M_3 \to 1$, we have $M_4 = \overline{M}_4$ and $(p_1M_1/p_4)^2 = \overline{(p_1M_1/p_4)}^2$.

It is clear from Fig. 3 that M_4 will be small when β is small. Even though both M_1 and M_4 are small, the flow is not necessarily incompressible, since M_3 may still be near unity. Figure 3 further shows (when $M_3 < 1$) that large values of M_4 cannot be attained for many porous media, since β is usually less than 0·6.

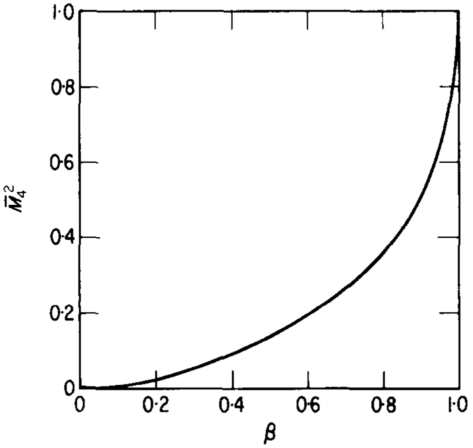

FIG. 3. $\overline{M}_4^2$ vs. β; $\gamma = 7/5$.

In the limit when *all* Mach numbers are small, we obtain from equations (2) the simple result

$$M_1 = \beta M_2 = \beta\left(\frac{p_4}{p_1}\right) M_3 = \left(\frac{p_4}{p_1}\right) M_4, \quad (5)$$

which is used later.

MOMENTUM

The foregoing analysis is completely independent of the momentum equation applicable between states 2 and 3. For compressible flow, this equation can be written as (see [4], Chapter 8)

$$dp + dX = -\rho u\, du. \quad (6)$$

The drag term is

$$dX = \tfrac{1}{2} b \rho u^2 dx, \quad (7)$$

where x is distance in the direction of flow, and b is a drag coefficient with dimensions (length)$^{-1}$, considered constant when any integration with respect to x is performed. Equation (7) is identical to that used in [4] (Chapter 6), with $b = (4f/D)$, and was chosen, in part, for this reason. A comparison with Darcy's law (as given, for instance, in [2], p. 2) shows that $b = (2\eta/\rho u B_0)$, where B_0 is Darcy's constant and η is the viscosity of the gas. A more detailed discussion of the physical implications of the drag coefficient is given in the last section.

Darcy's equation is universally used as the momentum equation for incompressible flow in a porous media. It is obtained by neglecting the convective term $\rho u\, du$ in equation (6) and integrating between 2 and 3, assuming the flow isothermal, to yield

$$bL = \beta^2 \frac{p_1^2 - p_4^2}{RT_1(\rho_1 u_1)^2} = \beta^2 \frac{1 - (p_4/p_1)^2}{\gamma M_1^2}, \quad (8)$$

where L is the thickness of the plate. In this case, only one constant, b/β^2, characterizes the gas–solid interaction. This is not surprising, since [6] develops a theory in which β does not enter explicitly. When compressibility becomes important this simplification is no longer possible and both b and β are separately significant parameters. Had the convective term been retained, equation (8) would have an additional $\ln (p_4/p_1)^2$ term on the right side [5]. This term, however, is small compared to the right side of (8), except for small values of $(p_4/p_1)^2$, when compressibility should not be neglected.

When equations (6) and (7) are integrated from 2 to 3 with the adiabatic assumption, we obtain

$$bL = F(M_2^2) - F(M_3^2), \quad (9)$$

where

$$F(M) \equiv \frac{1}{\gamma M^2} + \left(\frac{\gamma + 1}{2\gamma}\right) \ln \left[\frac{M^2}{1 + (\gamma - 1) M^2/2}\right],$$

which is shown in Fig. 4 for $\gamma = (7/5)$. For comparative purposes, this figure also shows $1/\gamma M^2$, which closely approximates F for small M^2.

By means of equations (5), equation (9) becomes for small Mach numbers

$$bL = \beta^2 \frac{1-(p_4/p_1)^2}{\gamma M_1^2} + \left(\frac{\gamma+1}{2\gamma}\right)\ln(p_4/p_1)^2, \quad (10)$$

which also differs from Darcy's equation by the logarithm term.

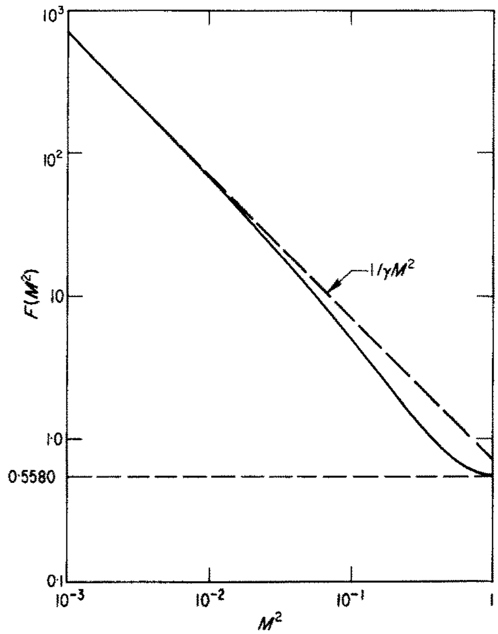

FIG. 4. $F(M^2)$ vs. M^2; $\gamma = 7/5$.

A simple criterion for the use of incompressible flow theory can now be derived. In doing so, we approximate F by $1/\gamma M^2$ and freely use equations (5). Since the largest of the Mach numbers is M_3 the flow is considered incompressible if $F(M_3^2) \geqslant 10$ or $0{\cdot}239 \geqslant M_3$ (see Fig. 4). From equation (9), we have

$$\frac{1}{\gamma M_3^2} = \frac{1}{\gamma M_2^2} - bL \geqslant 10,$$

which, after some manipulation, results in

$$(p_4/p_1)^2 \geqslant (p_4/p_1)_{\text{inc}}^2 = \frac{1}{1+(bL/10)}. \quad (11)$$

The flow is thus approximately incompressible if (p_4/p_1) is greater than $(p_4/p_1)_{\text{inc}}$.

Equations (2) and (9) show that

$$bL = f(M_1^2, \beta, \gamma, p_4/p_1). \quad (12)$$

An important conclusion based on equations (1) and (12) is that for given values of β, γ, p_4/p_1, and $p_1\rho_1$, the mass flow rate $\rho_1 u_1$ adjusts to conserve momentum. Any change in the flow rate through a plate thus requires a new p_4/p_1. For fixed values of β, γ, and p_4/p_1, an alternative interpretation is that momentum conservation relates in a single-valued manner M_1^2 and bL [5].

As noted earlier, the two Fanno curves coincide when $\beta = 1$. We therefore obtain the relation

$$f(M_1^2, 1, \gamma, p_4/p_1) = F(M_1^2) - F(M_4^2).$$

It is important, however, to note that [compare with equation (9)]

$$bL \neq F(M_1^2) - F(M_4^2),$$

when $\beta < 1$, since erroneous results are obtained if the surface effects are ignored [5]. This conclusion is evident from the fact that the average Mach number on the Fanno curve from 2 to 3 is greater than that on the one from 1 to 4.

CHOKED FLOW: $M_3 = 1$

When $(p_4/p_1) < \overline{(p_4/p_1)}$, equations (2c) and (3) no longer apply, and state 4 cannot be directly related to state 3 as before. These equations are replaced by $M_3 = M_{3'} = 1$, and conditions at 1, 2, and 3′ are unaffected by a further reduction in p_4.

The analysis is again conveniently illustrated on the Mollier diagram (Fig. 2). The limit $M_3 \to 1$, is shown as $\overline{4}$. For pressure ratios slightly less than $\overline{(p_4/p_1)}$, state 4 is to the right of $\overline{4}$ on the Fanno curve through state 1. State 4 thus moves in a continuous manner along the Fanno curve as p_4/p_1 decreases. If p_4 is further decreased, state 4 moves to 4* where $M_{4*} = 1$, and the corresponding pressure ratio is designated by $(p_4/p_1)^*$. A further decrease in p_4

results in supersonic flow, since $M_4 > 1$. For a sufficiently small pressure ratio, denoted by $\underline{(p_4/p_1)}$, conditions at 4 are reached by an isentropic expansion from 3′ to $\underline{4}$. Solutions do not exist for pressure ratios smaller than $\underline{(p_4/p_1)}$, which is referred to as the limiting pressure ratio.† This limit stems from our insistance on a constant-area duct. A smaller pressure ratio, for example, requires an expanding flow and an increasing duct cross-sectional area downstream of the plate.

It should be realized that the flow may be quite turbulent downstream of the plate. In this situation, state 4 is frequently downstream of the plate rather than at its surface.

Conditions at 4, when $\underline{(p_4/p_1)} \leqslant (p_4/p_1) \leqslant \overline{(p_4/p_1)}$ are determined by combining equations (2a, b) and (9) to yield

$$F(M_2^2) = bL + F(1), \tag{13a}$$

$$M_4^2 = \frac{1}{(\gamma - 1)} \times \{[1 + 2(\gamma - 1)(p_1/p_4)^2\beta^2 G_1(M_2^2)]^{\frac{1}{2}} - 1\}. \tag{13b}$$

Thus, M_4 depends only on γ, bL, and $(p_1\beta/p_4)$. The pressure ratio $(p_4/p_1)^*$ is obtained by setting $M_4 = 1$. The Mach number $\underline{M}_4$ at the limiting pressure ratio is determined by the isentropic relation

$$G_1(\underline{M}_4^2) = [2/(\gamma + 1)]^{(\gamma+1)/(\gamma-1)}\beta^2,$$

where equations (13), which still apply, can be solved for $\underline{(p_4/p_1)}$.

DISCUSSION

Three principal topics are discussed in this section, all of which bear on experimental verification. First, the various flow regimes are briefly described in terms of the pressure ratio. Next, an appropriate generalization of the flow rate vs. pressure differential plot is given. The third topic is a discussion of the physical implications of the drag coefficient b.

There are four flow regimes that can be characterized by the pressure ratio p_4/p_1. The first is $(p_4/p_1)_{\text{inc}} \leqslant (p_4/p_1) \leqslant 1$, where $(p_4/p_1)_{\text{inc}}$ is given by equation (11). This is the incompressible regime where Darcy's equation is valid. The next regime, $\overline{(p_4/p_1)} \leqslant (p_4/p_1) < (p_4/p_1)_{\text{inc}}$, is characterized by compressible subsonic flow everywhere. The third regime, $(p_4/p_1)^* \leqslant (p_4/p_1) < \overline{(p_4/p_1)}$, requires a choked flow, with subsonic flow at 4. The final regime, $\underline{(p_4/p_1)} \leqslant (p_4/p_1) < (p_4/p_1)^*$, also requires a choked flow, with supersonic flow downstream of the plate. Figure 5 shows these pressure

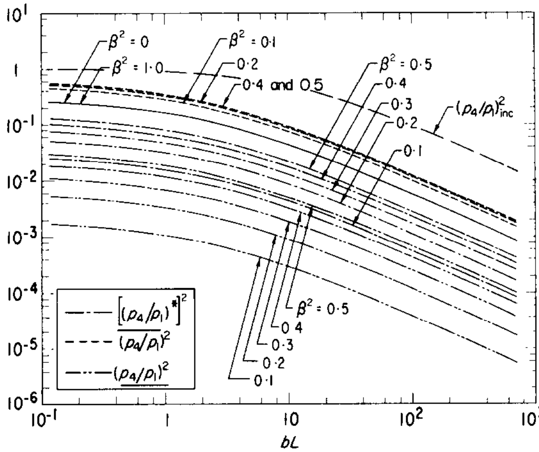

FIG. 5. $(p_4/p_1)^2_{\text{inc}}$, $\overline{(p_4/p_1)}^2$, $[(p_4/p_1)^*]^2$, and $\underline{(p_4/p_1)}^2$ vs. bL; $\gamma = 7/5$.

ratios as a function of bL for $\gamma = (7/5)$ and various β^2. The $\beta^2 = 1$ curve is common to all pressure ratios and is also the $\beta = 0$ curve for $\overline{(p_4/p_1)}^2$. All curves are parallel, except $(p_4/p_1)^2_{\text{inc}}$, basically because the same momentum equation (9) and Fanno curve equation (2a) apply to all of them. Values of (p_4/p_1) greater than $\overline{(p_4/p_1)}$ result in $M_3 < 1$; values less than $\underline{(p_4/p_1)}$ are not permissible. This figure demonstrates that the choking pressure ratio $\overline{(p_4/p_1)}$ is relatively insensitive to changes in β^2. We also observe that $(p_4/p_1)^*$ is more sensitive to β^2

† This limiting pressure ratio probably cannot be attained in practice due to the effects of turbulence downstream of the plate. A smaller downstream reservoir pressure than the limiting one should result in a flow at the end of the duct analogous to the flow external to an underexpanded nozzle.

than $\overline{(p_4/p_1)}$, while $\underline{(p_4/p_1)}$ is even more sensitive than $(p_4/p_1)^*$.

Of the four regimes, there can be no doubt about the first. The others still require experimental verification. This is particularly true of the last, since supersonic flow has not, as yet, been associated with flow through a porous medium. This regime may be the easiest to verify, however, since standard techniques, such as the Schlieren optical method, can be used to detect the presence of supersonic flow.

Incompressible flow through a porous medium is frequently represented by a plot of the volumetric or mass flow rate vs. the pressure or pressure-square differential. More appropriate for compressible flow would be a plot of M_1^2 vs. $(p_4/p_1)^2$, as shown in Fig. 6. [Once M_1 and (p_4/p_1) are known, M_4 is readily determined by equation (2a)]. This figure is for $\beta^2 = 0{\cdot}2$, $\gamma = (7/5)$, and various bL. The solid curves are for compressible flow, where the (△) denotes the pressure ratio $(p_4/p_1)_{\text{inc}}$, and the (○) denotes the onset of choking. These curves terminate at $\underline{(p_4/p_1)}$. The dashed line is the incompressible result, equation (10), while the doubly dashed line is Darcy's equation (8). For bL equal to 1 and 10, we see that the compressible and incompressible results depart at $(p_4/p_1)_{\text{inc}}$. For bL equal to 10^2 and $(p_4/p_1)^2 < 0{\cdot}1$, only small changes in M_1^2 are necessary for large changes in M_3 and p_4/p_1. Since this is true for both the

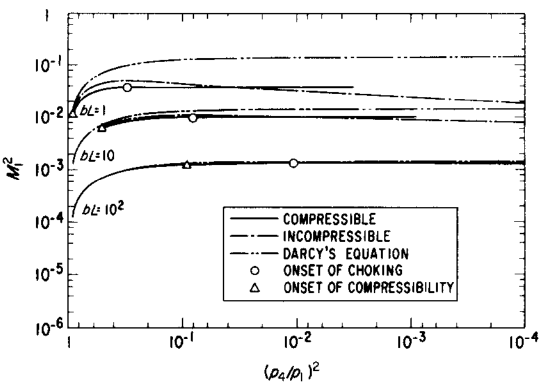

FIG. 6. M_1^2 vs. $(p_4/p_1)^2$ for $bL = 1, 10, 10^2$; $\beta^2 = 0{\cdot}2, \gamma = 7/5$.

incompressible and compressible results, the two remain quite close. For a thick plate, i.e. large bL, Darcy's equation adequately predicts M_1^2 vs. $(p_4/p_1)^2$. It does not, of course, correctly predict M_4 or $\underline{(p_4/p_1)}$.

For purposes of simplicity and clarity, only one constant, namely b, has been used in the momentum equation instead of two as in [3]. This is not an essential feature of this theory since much of it is given by the Fanno curves, which are independent of momentum considerations. Whether or not one or two constants are used, it is important to realize that they should account only for the dissipative mechanisms from 2 to 3. Dissipation due to surface effects should be dealt with separately.

Experimental data will not necessarily follow a single bL curve as flow conditions vary, since b generally depends on these conditions. In fact, part of the difficulty with b is that it depends on the gas, porous medium, and flow conditions. This is evident from the relation $b = (2\eta/\rho u B_0)$ given earlier.

The drag coefficient was introduced with the expectation that it might be independent of β, which must be specified as well. In addition, it is reasonable to expect that b does not depend on the molecular weight of the gas or on γ. The first exclusion stems from dimensional analysis, while the second is due to the explicit appearance of γ in the theory. In order to define more clearly the role of b, we proceed by comparing the flow in a porous medium with that in a pipe with roughened walls. This type of heuristic comparison is briefly pursued in order to introduce concepts that may be experimentally useful.

According to this analogy, b should depend on some average pore diameter d (or equivalently on the specific surface), and on the Reynolds number $R_d = (\rho_1 u_1 d/\eta_1)$. Thus, $b = (2d/B_0 R_d)$ and if d/B_0 is constant, b is inversely proportional to the Reynolds number, a result typical of laminar flows. For sufficiently large values of R_d, however, d/B_0 is not necessarily constant. The drag coefficient may also depend on the

Mach number, although this dependence might be weak (see [4], Chapter 6). For a specific porous medium, b may also change with time due to surface changes such as absorption. In addition, b may depend strongly on certain characteristics of the material, such as the relative roughness (see [4], Chapter 6). For sufficiently large flow rates b may be nearly independent of R_d [4]. In this circumstance, b may depend primarily on a relative roughness. This regime should be useful for experimentally verifying this analysis.

Even if b does not have precisely the behavior described above, experimental verification is still possible. For example, use can be made of the fact that once choking has occurred, the flow from 2 to 3′ does not change and bL remains constant. Furthermore, in the compressible regime when $M_3 < 1$, problems associated with b might be avoided by using different plate thicknesses of the same porous material. For engineering purposes, plots of b vs. the pertinent parameters, such as R_d, are of course necessary.

In conclusion, we note that quantities other than the pressure and Mach number are easily determined. For example, the temperature at 4 is

$$\frac{T_1}{T_4} = 1 + \left(\frac{\gamma - 1}{2}\right) M_4^2,$$

since the stagnation temperature, approximately given by T_1, is a constant. Another useful quantity is the stagnation pressure p_0. Upstream of the plate it is approximately equal to p_1, but at 4 it is

$$\frac{p_{04}}{p_4} = [1 + (\gamma - 1) M_4^2/2]^{\gamma/(\gamma-1)},$$

and consequently, $(p_4/p_1) < (p_{04}/p_{01})$. This difference in static and stagnation pressure ratios is not necessarily small, e.g. with $\gamma = (7/5)$ and $M_4 = 1$ we have $(p_{04}/p_{01}) = 1{\cdot}892\ (p_4/p_1)$.

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One reviewer has kindly pointed out two recent publications related to this work. The first one, by Pinker and Herbert [7], deals with the pressure loss through a single wire mesh gauze. The second one is an abstract by Shreeve [8] which appears to verify experimentally our analysis.

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Résumé—Une théorie unidimensionnelle simple est donnée pour l'écoulement permanent, compressible et adiabatique d'un gaz parfait à travers une plaque poreuse. La relation de Dupuit–Forcheimer, valable pour l'écoulement incompressible, est remplacée par un processus d'élargissement brusque isentropique lorsque celui-ci existe. On emploie l'équation de Darcy sous une forme applicable à l'écoulement adiabatique compressible. Une conséquence de cette étude est que le nombre de Mach à la surface aval peut être beaucoup plus faible que l'unité, même lorsque l'écoulement à travers la plaque est bloqué. Lorsque la pression sur la plaque diminue, l'ecoulement reste bloqué, mais le nombre de Mach aval augmente. Pour une pression aval suffisamment faible ce nombre de Mach sera supérieur à l'unité. Une large gamme de nombres de Mach aval à partir du subsonique est ainsi possible, même si l'écoulement est bloqué. Pour un écoulement compressible, le débit volumique est déterminé habituellement par la différence de pression à travers la plaque. On montre que la relation compressible équivalente consiste à porter le nombre de Mach amont en fonction du rapport de pression à travers la plaque. Le résultat incompressible peut également être porté sur ce diagramme; il diffère du résultat compressible, sauf lorsque la plaque est épaisse.

Zusammenfassung—Für die stationäre, kompressible, adiabate Strömung eines idealen Gases durch eine poröse Platte wird eine einfache, eindimensionale Theorie angegeben. Die Dupuit–Forchheimer–Beziehung, die für inkompressible Strömung gültig ist, wird durch Beziehungen für eine isentrope Kompression beim Eintritt in die Platte und für einen nichtisentropen, plötzlichen Erweiterungsvorgang beim Austritt aus der Platte ersetzt. Die Gleichung von Darey wird in einer Form verwendet, die für eine adiabate, kompressible Strömung anwendbar ist. Ein Ergebnis dieser Untersuchung ist die Tatsache, dass die Machzahl hinter der Platte viel kleiner als 1 sein kann, sogar, wenn die Strömung durch die Platte maximal möglichen Massendurchsatz hat. Wenn der Druck an der Plattenrückseite abnimmt, hat die Strömung weiterhin maximal möglichen Massendurchsatz, die Machzahl hinter der Platte steigt jedoch an. Für einen genügend kleinen Druck hinter der Platte wird diese Machzahl grösser als 1. Es ist also ein grosser Bereich von Machzahlen zwischen Unterschall und Überschall möglich, obwohl die Strömung maximal möglichen Massendurchsatz hat. Für eine inkompressible Strömung wird der Volumenstrom gewöhnlich durch den Druckabfall an der Platte bestimmt. Die entsprechende Beziehung für kompressible Strömung wird als Diagramm dargestellt, in dem die Machzahl vor der Platte über dem Druckverhältnis an der Platte aufgetragen ist. Das Ergebnis für den inkompressiblen Fall kann in diesem Diagramm ebenfalls dargestellt werden; ausser für dicke Platten unterscheidet es sich von dem für den kompressiblen Fall.

Аннотация—Приводится простая одномерная теория для стационарного сжимаемого адиабатического течения идеального газа через пористую пластину. Уравнение Дюпи-Форшеймера, справедливое для несжимаемого потока, заменено соотношениями для изентропического процесса внезапного расширения, когда такой существует. Уравнение Дарси используется в виде, приемлимом для сжимаемого адиабатического потока. Из данного исследования вытекает, что число Маха на расположенной вниз по потоку поверхности может быть гораздо меньше единицы даже, елси происходит запирание потока через пластину. При уменьшении давления на пластине поток остается запертым, а число Маха вниз по потоку увеличивается .Для достаточно малого давления число Маха будет гораздо больше единицы. Поэтому может иметь место широкая область изменения чисел Маха вниз по потоку от единицы даже при запирании потока. В случае несжимаемой жидкости объемный расход обычоно определяется разностью давления поперек пластины. Показано, что эквивалентное соотношение для случая сжимаемого газа показано на графике зависимости числа Маха вверх по потоку от перепада давлений поперек пластины. На данном графике можно также показать результата для несжимаемого потока; они отличаются от результатов для сжимаемого, за исключением случая толстой пластины.